

# SIMULTANEOUS PROPAGATION OF BOUND AND LEAKY DOMINANT MODES ON PRINTED-CIRCUIT LINES: A NEW GENERAL EFFECT

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## Abstract

We were the first to report (at the 1993 IEEE International Microwave Symposium) that both the bound and leaky dominant modes can propagate **simultaneously** on conductor-backed coplanar strips over a frequency range. We have recently studied this interesting and initially unexpected effect in more detail, and we have made two important discoveries:

(1) The simultaneous-propagation effect can actually occur on **most** printed-circuit transmission lines (its presence depending on the relative line dimensions), so that, contrary to earlier belief, the effect is rather **general**.

(2) We have discovered the surprising presence of a **new improper** (or nonspectral) **real solution**, which is **nonphysical** but whose evolution as a function of dimensional change serves to **explain** how the simultaneous-propagation effect can occur. The new solution, and its behavior in a completely nonphysical region, thus govern otherwise-mysterious large changes in the physical, measurable solutions.

## I. THE EFFECT AND ITS GENERALITY

Let us consider these two points (the generality and the explanation) separately.

Under usual circumstances for printed-circuit lines, the guided dominant mode is purely bound at lower frequencies and then becomes leaky above some critical frequency. This critical frequency occurs when the dispersion curve for the dominant mode crosses the dispersion curve for the surface wave (or parallel-plate mode) into which the leakage occurs. At this crossing, we always find some complicated fine structure and a small range in frequency within which the solution for the dominant mode is nonphysical. We have called this small range a "spectral gap." In the usual situation, therefore, we find that the bound dominant mode can be observed only **below** the frequency at the spectral gap

sets in, and the leaky dominant mode propagates only **above** the frequency at which the spectral gap ends, so that two solutions are **completely separated** from each other.

As an example of printed-circuit lines, let us consider here the conductor-backed coplanar strips shown in the inset of Fig.1. Figure 1 itself shows clearly that the bound and leaky portions of the dominant mode are indeed separated from each other, in accordance with the comments in the previous paragraph. The pattern of the spectral gap between the bound and leaky portions is of the standard form, although it is difficult to observe it in Fig.1 because of the compressed scale. The strip width for the results in Fig.1 is relatively **narrow**, with  $w/h = 0.25$ , where  $w$  is the strip width and  $h$  is the substrate height.

When the relative strip width is **increased** to  $w/h = 0.60$ , as shown in Fig.2, the spectral gap disappears, and both the bound and leaky modes are present simultaneously in the frequency range between  $f_{cr1}$  and  $f_{cr2}$ . The overlap region, in which both mode types appear simultaneously, becomes even greater when  $w/h$  is increased further.

Results for this printed-circuit transmission line were presented by us in [1]. Since then we have found that the same effect (that is, the change from separate bound and leaky dominant-mode regions to a range in frequency where they are present simultaneously) occurs on many kinds of printed-circuit lines, with both isotropic and anisotropic substrates. This effect was thought by us originally to be a rarity, but we now know that it is quite **general**. A recent interesting paper [2], following [1], demonstrates similar behavior for **coupled slot lines**, where the pair of slots may be viewed as a sort of dual to the pair of strips in our case of conductor-backed coplanar strips. We will present results here for a **single** slot line, to illustrate that similar results can indeed occur on a quite different type of line.

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The **slot line** for which numerical calculations will be presented is the conventional (not conductor-backed) line with a single slot and infinitely wide metal side strips. We begin with a relatively narrow slot, with  $w/h = 0.4$  and  $\epsilon_r = 2.25$ , where  $w$  is the slot width (in Figs.1 and 2  $w$  was the strip width) and  $h$  is the height of the dielectric layer. The dispersion results for this case are shown in Fig.3. We observe that the dispersion behavior in Fig.3 is very similar to that shown in Fig.1 for the conductor-backed coplanar strips, where the bound and leaky modes are completely separated from each other. Such behavior is what we now know to expect in most cases. As we increase the ratio  $w/h$ , widening the slot, the spectral gap changes its nature, so that when we finally reach  $w/h = 1.0$ , as in Fig.4, the dispersion behavior becomes completely different in that frequency range. We note in Fig.4 that, as in Fig.2, the spectral gap has disappeared, and that the bound and leaky modes now are present simultaneously over a frequency range.

The behavior shown in Fig.4 has not been known previously for slot lines, and it may come as a surprise. We should note, however, that it occurs only for relatively wide slots, wider than those customarily employed. It is important to recognize, however, that such very different dispersion behavior can result from modifying **only** the relative dimensions of the line.

## II. THE NEW IMPROPER REAL SOLUTION: ITS EVOLUTION AND ROLE IN EXPLAINING THE EFFECT

Let us first look at Fig.1, which holds for conductor-backed coplanar strips, and applies to narrow strips, for which the bound and leaky modes are completely separated. In the bound-mode region, we see that there are two solutions, the actual bound mode, which is real and proper, and an improper real mode, which is nonphysical. In the leaky-mode region, we have the leaky mode and also its complex conjugate, which is nonphysical (since its  $\alpha$  value increases as the wave propagates) and also has the same value of  $\beta$  as the leaky mode so that we cannot see it separately. Thus, we have **two** solutions at **all** frequencies.

When we look at Fig.2, however, for which we find that the bound and leaky modes are present simultaneously over a frequency range, we notice that there are **three** solutions at all frequencies. Again we must recognize that the leaky solution has its complex conjugate, and that for very low frequencies there are two improper real solutions. This change in the number of solutions appears mysterious, and it contradicts what we expect from analytical continuity; a solution in Fig.1 should migrate continuously to a corresponding one in Fig.2 at each frequency, as the  $w/h$  value is increased continuously. However, we do not find that this is the case when we inspect these two figures.

To explain this mysterious behavior, we intuitively presumed that there should be a solution that is presently missing at all frequencies in Fig.1. We therefore searched for such a new solution in the region of  $\beta/k_0 > (\epsilon_r)^{1/2}$  when  $w/h = 0.25$ , and found, as we expected, a **new improper real** solution which lies in the region  $\beta/k_0 > 1.5$  in the frequency range of Fig.1. This is a **nonphysical** region in which no one would ordinarily look. This discovery of the new solution provides the completeness needed in our discussions; the dispersion structure in Fig.1 now has **three** solutions at each frequency, and the mystery in the number of solutions is removed. More important, this completeness aspect greatly helps to understand the migration of solutions in the **physical** region, relating to the simultaneous-propagation effect.

The new improper real solution, which is nonphysical everywhere, lies far above the other solutions shown in Fig.1. As  $w/h$  is increased, the new solution drops down, and the other improper real solution, shown as a dot-dash curve, peaks upward to meet the new downward-coming solution. At  $w/h = 0.370$ , as seen in Fig.5, these two curves almost touch each other; this interaction occurs in the vicinity of  $\beta/k_0 = (\epsilon_r)^{1/2}$ . As  $w/h$  is increased only slightly further, the two curves touch and then pull away from each other in a direction at right angles to the original approach, producing separate low-frequency and high-frequency portions. When they pull away, however, the gap between them is not empty but contains a **new improper complex (leaky) solution**, as seen in Fig.6 between points 1 and 2. Note that in Fig.6  $w/h = 0.372$ , which is only a very slight increase from the value in Fig.5.

As  $w/h$  is increased, the width of the interaction gap increases; accordingly, point 1 moves upward, to lower frequencies, while point 2 moves downward, to higher frequencies. For still further increases in the  $w/h$  value, point 2 moves down until it meets point 3, at which time the new complex (leaky) solution becomes **continuous** with the original leaky solution. For that case, however, the spectral gap seen in Fig.1 is still present, and there is no significant change yet in the dispersion behavior of the solutions in the physical region. As  $w/h$  is increased somewhat more, the leaky solution moves down further and crosses the proper real solution at a lower frequency so that the spectral gap disappears, and the situation of simultaneous propagation illustrated in Fig.2 results.

An exactly analogous situation occurs for the slot line geometry for which the dispersion behavior is shown in Figs.3 and 4 for narrow and wide slots, respectively. For this case we also found an additional solution which lies high up (off the scale) in Fig.3, and then saw that the evolution of solutions, as  $w/h$  is increased, follows precisely the evolution described above, ending with the simultaneous-propagation effect

seen in Fig.4. Similar results were found for other printed-circuit transmission-line geometries on both isotropic and anisotropic substrates, and also when the strips in coplanar waveguide or coplanar strips are unequal.

The talk will include more of the various intermediate stages in the evolution from Fig.1 to Fig.2 and from Fig.3 to Fig.4.

### III. CONCLUSIONS

The effect we are discussing is that, by changing *only* the relative dimensions of a printed-circuit transmission line (such as a strip width or a slot width), the dispersion behavior for the dominant mode can change drastically, from the expected case for which the bound-mode and the leaky-mode regions are *completely separated* from each other to an unexpected situation in which the bound and leaky modes can propagate *simultaneously* over a frequency range, which in fact can be very large. The practical importance of this result is that, if the transmission-line circuit is designed on the assumption that only the bound mode is present, but unexpectedly a leaky mode is there as well, the source designed to excite the bound mode will excite the leaky mode also, and with comparable amplitude because both modes have similar strip current distributions. The circuit will then suffer from unexpected cross talk and power loss.

When we originally found this simultaneous-propagation effect on conductor-backed coplanar strips, we believed that the phenomenon was a rarity. We have

since discovered that this basic effect is quite *general*, and that it occurs on most printed-circuit transmission lines, for both isotropic and anisotropic substrates.

In this study, we also discovered the existence of a *new improper real mode*, which is *always nonphysical* but which serves to explain how the transition from separate regions to simultaneous propagation can occur. An examination of the evolution of this new solution as the relative line dimensions are modified shows that rather complicated changes occur in the nonphysical region and that the new solution plays a key role in them. It is particularly interesting that the complicated behavior in the nonphysical region ultimately leads to the important changes in the physical region discussed here.

### ACKNOWLEDGMENT

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### REFERENCES

- [1] H. Shigesawa, M. Tsuji and A. A. Oliner, "Simultaneous propagation of both bound and leaky dominant modes on conductor-backed coplanar strips," 1993 IEEE/MTTS Intern'l Microwave Symp. Digest, pp.1295-1298, Atlanta, GA, 1993.
- [2] Y. -D. Lin and Y. -B. Tsai, "Surface wave leakage phenomena in coupled slot lines," IEEE Microwave and Guided Wave Lett., vol.4, pp. 338-340, Oct. 1994.

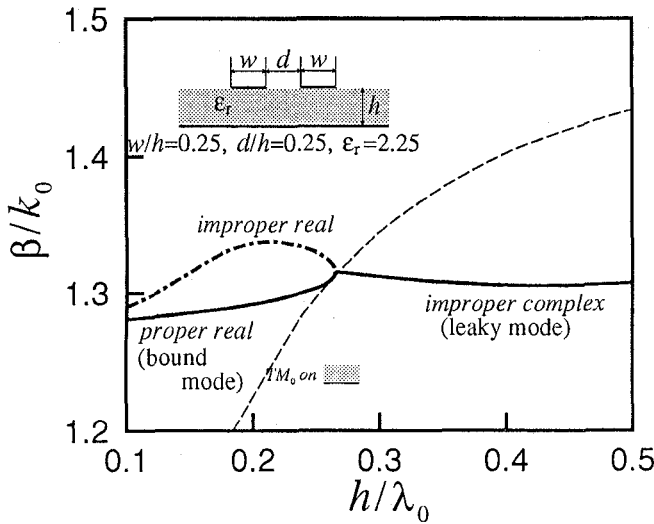


Fig.1. The normalized phase constant  $\beta/k_0$  for  $w/h=0.25$  (narrow strips) as a function of  $h/\lambda_0$  for conductor-backed coplanar strips. The bound-mode and leaky-mode regions are completely separated from each other, and a spectral gap is present between them.

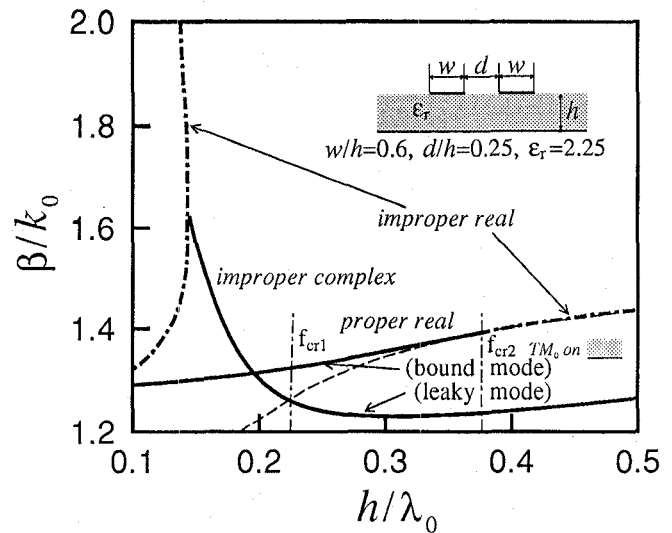


Fig.2. When  $w/h=0.60$  (wider strips) in the conductor-backed coplanar strips, the spectral gap disappears, and both the bound and leaky modes are present simultaneously in the frequency range between  $f_{cr1}$  and  $f_{cr2}$ .

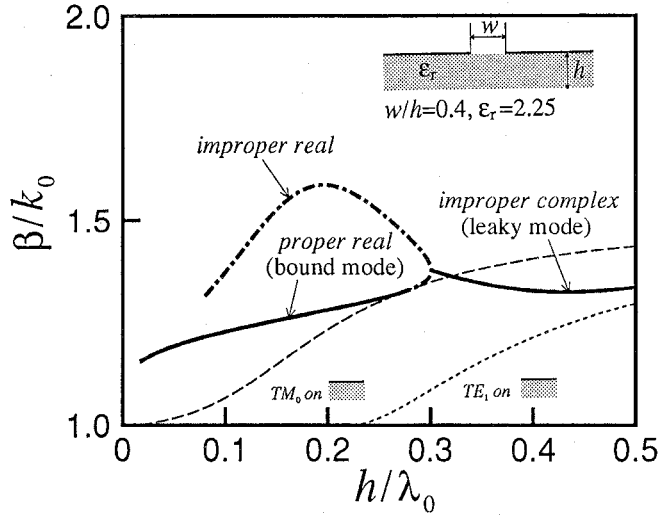


Fig.3. A plot similar to that in Fig.1, but for a **narrow** slot,  $w/h=0.40$ , on conventional slot line. The dispersion behavior is similar to that seen in Fig.1.

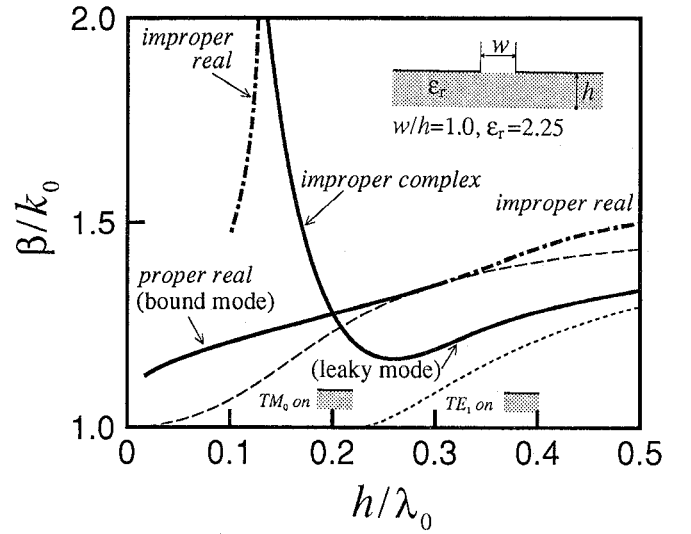


Fig.4. A plot similar to that in Fig.2, but for conventional slot line with a **wide** slot,  $w/h=1.0$ . The dispersion behavior is similar to that in Fig.2, where the bound and leaky modes are present simultaneously over a frequency range.

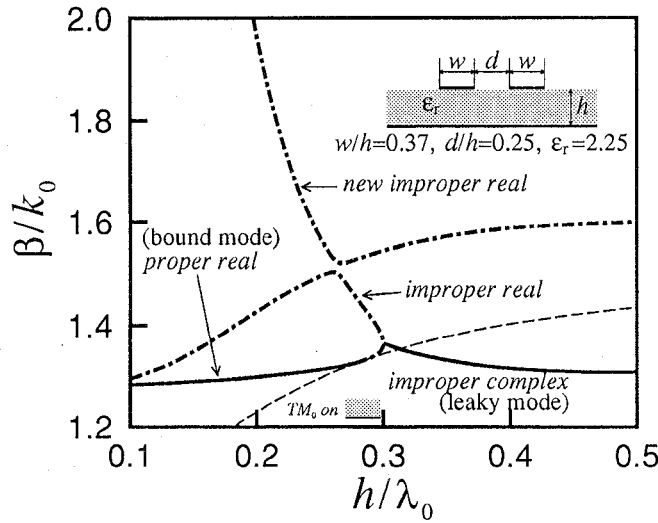


Fig.5. The dispersion behavior for conductor-backed coplanar strips with an **intermediate** value of strip width,  $w/h=0.370$ . The new improper real solution is seen to interact strongly with the original one at around  $\beta/k_0=[\epsilon_r]^{1/2}$ .

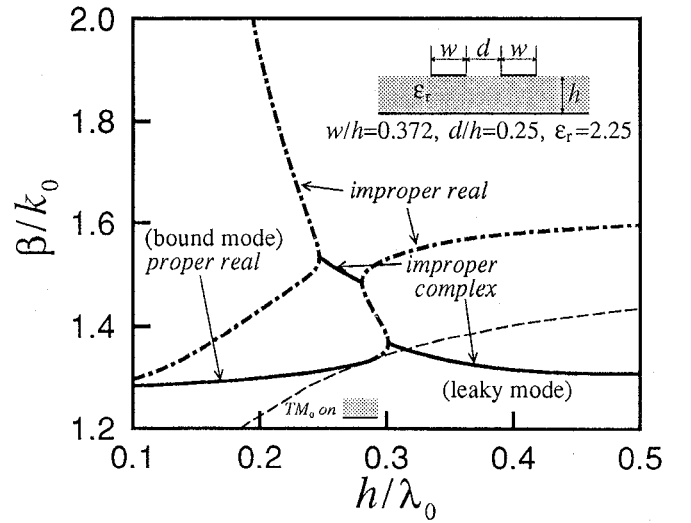


Fig.6. The strip width  $w/h=0.372$  has been increased only very slightly from the value in Fig.5. It is seen, however, that the improper real solutions change dramatically, and that a new complex solution emerges in the interaction gap. The growth of the complex solution in the interaction gap eventually contributes to the simultaneous propagation.